Physics IV
ISI B.Math
Midterm Exam : March 3,2015

Total Marks: 60
Time : 3 hours
Answer all questions

1. (**Marks: 2 + 2 + 2 + 2 + 2 + 2 = 12**)

State whether the following are true or false with a one or two line justification :

(i) Two clocks at the extreme ends of a train are synchronized with respect to the train frame. You are standing on the ground and the train moves past you. To you, the clock at the front of the train will show a later time compared to the clock at the back.

(ii) An observer in Delhi reports a bomb explosion at 12 pm and another observer reports a bomb explosion in Kolkata at 2pm. It is not possible to find an inertial reference frame where the two explosions occur at the same time.

(iii) An observer in a certain inertial frame S detects and measures an electric field but no magnetic field produced by a certain charge configuration. It is always possible to find another inertial frame moving with an appropriate uniform velocity with respect to S such that an observer in this frame will detect a magnetic field but no electric field produced by the same charge configuration.

(iv) The total energy of an isolated electron at rest is zero

(v) A heavy particle at rest decays into two lighter particles. The mass of the heavy particle must be equal to the sum of the masses of the decay products.

(vi) Isaac carries two firecrackers A and B . He stands in a spot and lights A at 4pm and lights B at 4:05 pm. It is possible for Albert running past Isaac at uniform velocity to see B flash first and A flash later if he runs at an appropriate speed .

2. (**Marks : 2 + 2 + 2 + 2 + 2 + 2**)

Draw the ct and x axis of the spacetime coordinates of the observer $\mathcal{O}$. Then draw

(a) the world line of $\mathcal{O}$'s clock at $x = 1m$.

(b) ct' and x' axes of an observer $\mathcal{O}'$ who moves with a velocity $v = 0.5c$ in a positive x direction relative to $\mathcal{O}$ and whose origin ($x' = ct' = 0$) coincides with that of $\mathcal{O}$.

(c) the locus of events whose spacetime interval $(\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2$ from the origin = - 1 m^2

(d) the locus of events whose spacetime interval $(\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2$ from the origin = 0.

(e) the locus of events, all of which occur simultaneously at $ct = 2m$ according to $\mathcal{O}$.

(f) the locus of events, all of which occur simultaneously at $ct' = 2m$ according to $\mathcal{O}'$.

3. (**Marks : 3 + 3 + 3 + 3 = 12**)

(a) Show that the the sum of two future-pointing null four-vectors is either timelike or null, and is again future-pointing. Under what conditions is the sum null?

(b) Every four-vector orthogonal to a timelike vector is spacelike. (Two four vectors A^μ and B^μ are orthogonal if $A^\mu B_\mu = B^\mu A_\mu = 0$ $\mu = 0, 1, 2, 3$)

(c) If P^μ is the four-momentum of a particle and F^μ is the four-force defined as the product of the mass and the four-acceleration , show that $P^\mu F_\mu = 0$.

(d) Using the result in (c) , show that the four force F can be written as $F = (\frac{\gamma}{c}\mathbf{f}\cdot\mathbf{v}, \gamma\mathbf{f})$ where $\mathbf{v}$ is the three-velocity of the particle and $\mathbf{f}$, the three force $= \frac{d\mathbf{p}}{dt}(\mathbf{p} = m\gamma\mathbf{v}$. $\mathbf{p}$ being the relativistic three momentum).

4. (**Marks : 3 + 3 + 3 + 3 = 12**)

A train of proper length L moves at speed v_1 with respect to the ground. A passenger runs from the back of the train to the front with a speed v_2 with respect to the train. What is the distance covered and the time taken by the passenger

(i) In the train frame ?

(ii) In the ground frame ?

(iii) In a frame of the passenger ?

(iv) Verify that the invariant interval is the same in all frames

5. (**Marks: 5 + 2 + 5**)

(a) Show that it is impossible for an isolated free electron to emit or absorb a photon. [Hint: Imagine that this process is possible and then show that it contradicts energy and momentum conservation.]

(b) Write down a relativistic expression for the kinetic energy of a particle of mass m and speed v. Show that it reduces to the usual Newtonian expression in the appropriate limit.

(c) The electric and magnetic fields $(\mathbf{E}, \mathbf{B})$ are measured with respect to an observer in an inertial frame S .The corresponding fields $(\mathbf{E}', \mathbf{B}')$ are measured by another observer in frame S' which is moving at a velocity $\mathbf{v}$ along the common $x - x'$ axis with respect to the S frame. Show that $\mathbf{E}\cdot\mathbf{B}$ is an invariant quantity under Lorentz transformations. A particular electromagnetic field has its $\mathbf{E}$ field at an angle θ_0 to its $\mathbf{B}$ field, and θ_0 is invariant to all observers. What is the value of θ_0 ?